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*Synopsis of*

**Heat Transfer Enhancement in  
Microchannel Heat Sink using Passive  
Techniques for Electronic Cooling  
Applications**

*A Thesis*

*To be submitted by*

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*For the award of the degree*

*of*

**DOCTOR OF PHILOSOPHY**

# 1 Abstract

The emerging field of electronic industry requires compact electronic components without sacrificing the reliability and performance of the equipment. Miniaturization of electronic components requires compact and effective cooling techniques to dissipate large heat flux without a significant increase in pumping power. Under continuous operation, the temperature of the components may exceed the permissible limit due to poor heat dissipation. Single phase liquid cooling is preferred for high heat dissipation compared to conventional air cooling. Microchannels are used because of high surface area to volume ratio, compactness and efficiency. Microchannel heat sink with liquid as working fluid is a suitable technique for the purpose. Three dimensional thin-walled rectangular microchannel with fixed aspect ratio and hydraulic diameter is considered to analyse the sole effect of the proposed microchannel configurations. The improvement in hydro-thermal performance of conventional plane microchannel heat sink is analysed by introducing surface modifications, solid and porous inserts. The effects of conjugate heat transfer have been studied for different materials and various bottom substrate thicknesses. The proposed microchannel configurations were chosen based on effective heat transfer enhancement, requirement of minimum pumping power and ease of manufacturing. The geometry creation and mesh generation are carried out using Autodesk Inventor and ICEM CFD, respectively. Three-dimensional numerical simulations are carried out using Finite Volume Method based solver, ANSYS Fluent (flow and heat transfer simulation software). The study aims to enhance the heat transfer by breaking the thermal boundary layer and decreasing the convective thermal resistance with small increase in pumping power. The effect of surface modification on the flow characteristics is carefully studied by allowing fully developed flow at the channel entry corresponding to flow Re from 100 to 1000. The computational results were compared and validated with existing correlations and experimental results available in literature. The computational results are presented in the form of Nusselt number, pressure drop, friction factor, velocity and temperature distributions. The entropy generation minimization principle, field synergy number and performance index are the parameters used to quantify the overall performance improvement in the proposed microchannels heat sink. The heat transfer analysis showed reduction in overall thermal resistance, improvement in heat transfer as well as uniform surface temperature distribution with associated increase in pressure drop.

## 2 Research Gap

Geometry and surface modification are the two major components that significantly affect the performance of microchannels. These modifications include channel cross-section, wall waviness in the channel (Yuan *et al.* (2020); Xie *et al.* (2013)), fins (Jia *et al.* (2018)), ribs and cavities (Ghani *et al.* (2017)), wire coils (Feng *et al.* (2017)) and dimples in the microchannel (Wei *et al.* (2007)). But the heat transfer enhancement is at the expense of additional pumping power. The present study provides the optimum design of microchannel by analysing the trade-off between heat transfer and pressure drop. Electronic components generate heat at different regions and the temperature distribution on the surface of the heat sink is not uniform. A simple plane microchannel heat sink is not effective when these heat sources are placed under the heat sink. Such practical condition is taken into account in the present study by imposing partly heated

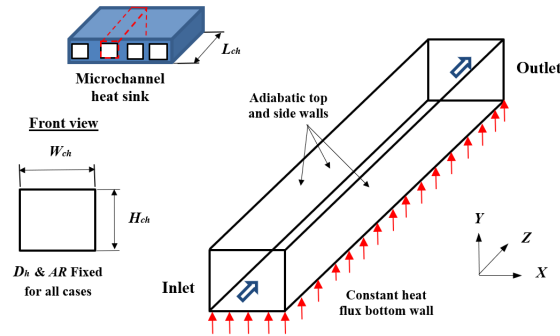


Figure 1: Computational Domain - Simple Microchannel

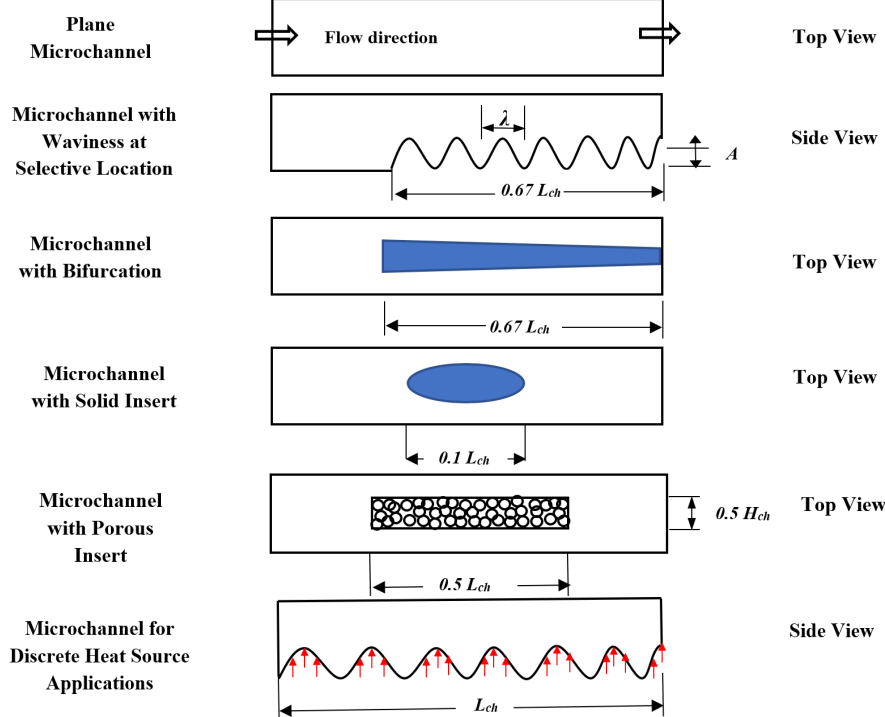


Figure 2: Microchannel with Passive Techniques

regions at the bottom wall. Microchannels with wavy bottom wall used in the present analysis to meet the above requirement because many studies (Mohammed *et al.* (2011); Rostami *et al.* (2015); Chiam *et al.* (2016)) were focused on providing waviness in the side walls of microchannels.

Manufacturing a metal microchannel with variable height micro-pillars, fins, protrusions is challenging. Therefore, the present study proposes the analysis of microchannel with inserts height equivalent to channel height. The straight and wavy tape inserts can be mounted in the channel that would act as bifurcation Feng *et al.* (2020). Based on manufacturing point of view, only single insert or bifurcation plate is used in the present study. It would lead to more unfavorable pressure drop with increasing stages of bifurcation downstream as it was also reported in Xie *et al.* (2014). Studies related to porous microchannel heat sink are very scarce. Shen *et al.* (2017); Dey and Saha (2021) studied microchannels with metal foams and fish-scale structures but there was not enough discussion on slip effect and porous media properties. The present study focuses on the combined effect of porous medium and flow bifurcation in the microchannel heat sink in order to avail the combined advantage of large convective area, fluid mixing and redevelopment/restart of thermal boundary layer.

### 3 Objectives

The main objectives of the present study are

- To validate the present numerical results with experimental results for the case of simple plane microchannel.
- To optimize the wavelength and amplitude of wavy wall present in microchannel for maximum heat transfer and minimum pressure drop.
- To analyse the effect of different working fluids on flow and heat transfer characteristics.
- To study the effect of substrate material and thickness on thermal performance of microchannel heat sink.
- To study the effect of various new passive heat transfer techniques as applicable to microchannel. The techniques include channel bifurcation, solid inserts and porous fins.
- To analyze the overall performance of proposed microchannel configuration through performance index and entropy generation minimization principle.
- To identify suitable microchannel heat sink configuration to achieve almost uniform temperature distribution for partly heated region application.

### 4 Most Important Contributions

The improvement in thermal performance of simple microchannel heat sink (Fig. 1) has been achieved by many researchers through introducing active/passive techniques and different working fluids. Each and every technique has both advantages and disadvantages but when designing a liquid cooled microchannel heat sink the foremost important thing is requirement of pumping power and manufacturing feasibility. With this in mind, the novel microchannel configurations were proposed in this study as shown in Fig. 2. The proposed microchannel heat sinks can enhance heat transfer with less pressure drop and maintains almost uniform temperature distribution.

The geometry creation and unstructured mesh generation has been done using Autodesk Inventor software and ANSYS ICEM CFD software tool respectively. Each proposed microchannel configurations have unique design like microchannel with solid, porous inserts and waviness. Therefore, the grid size will vary depend on location of waviness/microstructures. For example, the optimum grid size for microchannel heat sink with wavy bifurcation case were chosen from the grid independence study as  $40 \times 30 \times 600$ . Numerical Solutions has been done with Finite Volume Method based solver ANSYS Fluent-19. Semi-Implicit Pressure Linked Equation (SIMPLE) scheme is used for solving pressure-velocity coupling and both momentum and energy equations are solved using the second-order upwind scheme. The convergence criteria used for the present analysis is  $10^{-6}$ .

The microchannels used in the present study are easy to manufacture. Based on a thorough literature study, the rectangular cross-sectional microchannel considered throughout the present study and in order to find the sole effect of surface/wall modifications in the channel, the fully developed velocity profile is assigned at the inlet. Because the proposed modifications in simple microchannel heat sink will be at certain locations in the flow path. The conjugate analysis has been done only for wavy bottom wall microchannel to know the effect of different substrate material and thicknesses. The other common assumptions are steady, laminar fluid flow and without considering radiation effects. The pressure drop in microchannel flow is large, and leads to viscous

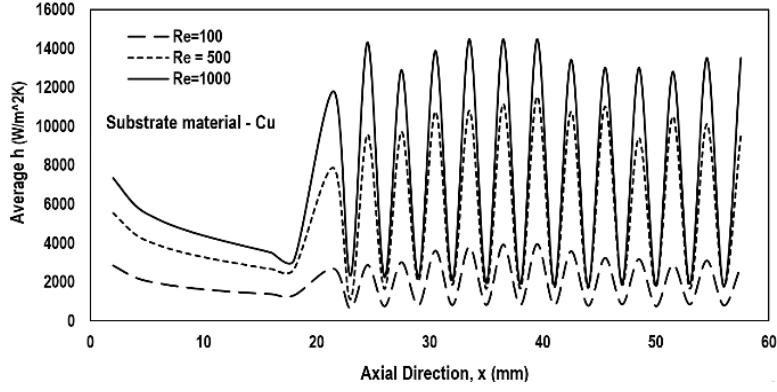


Figure 3: Axial Variation of Heat Transfer Coefficient in Wavy Wall Microchannel

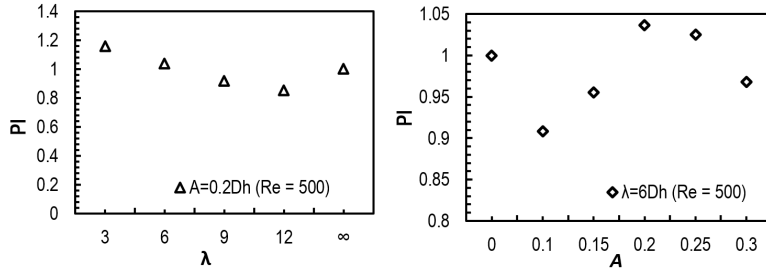


Figure 4: Effect of  $\lambda$  and  $A$  on Performance Index

dissipation. However, internal heating due to viscous dissipation will be substantially lower in liquids compared to air due to the large heat capacity of liquids relative to air. (Sharp *et al.* (2005)). Mainly water is considered as a working fluid in the present work. Further, the effect of nanofluids is also studied. The highlights of present study and its novelty are given below.

#### 4.1 Microchannel Heat Sink with Waviness at Bottom Wall

The waviness at selective location meets both the objectives of maximum heat transfer enhancement and minimum increase in pressure drop. It is observed that only few studies are available related to microchannel heat sink with waviness at bottom wall. The location of the waviness has been chosen based on the axial variation of heat transfer coefficient in the simple plane microchannel (no waviness).

The thermal performance improvement in the proposed wavy microchannel has been observed through axial variation of average heat transfer coefficient which shows greater values than plane microchannel as shown in Fig. 3. Based on the obtained numerical results the following  $Nu$  relation (Eqn. 1) is obtained from curve fitting. The correlation is the function of  $Re$ , wavelength ( $\lambda$ ) and wave amplitude ( $A$ ). The numerical results match with proposed correlation for all wavy channels. The maximum value of performance index (Eqn. 2) is at WC-1 ( $A = 0.2D_h$  and  $\lambda = 3D_h$ ). If wavelength increases the performance index value decreases for fixed value of  $A$ . But there is decreasing-increasing trend in performance index values when  $A$  value increases with constant  $\lambda$  as shown in Fig. 4.

$$Nu = \left[ \frac{-0.6049}{10000} Re^2 \right] + \left[ \frac{16.5157}{100} Re \right] + [2.5238 Re^{0.1} K^{-0.41}] - [11.8276] \times K^{0.41} \quad (1)$$

where

$$K = \frac{(A/D_h)^{2.1452}}{(\lambda/D_h)}$$

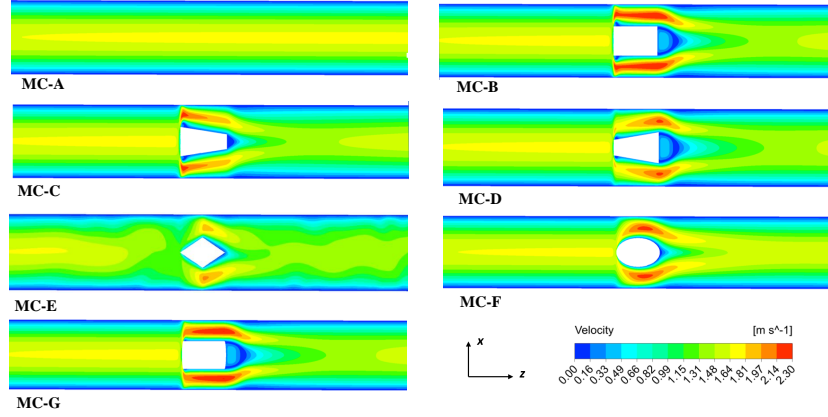


Figure 5: Effect of Insert on Velocity Distribution at  $x - z$  Mid-Plane ( $Re = 800$ )

$$PI = \frac{\left(\frac{Nu}{Nu_0}\right)}{\left(\frac{f}{f_0}\right)^{1/3}} \quad (2)$$

## 4.2 Microchannel Heat Sink with Solid and Porous Inserts

From the literature review, it is clear that the solid inserts in the flow path of the channel is able to increase the thermal performance. Although few studies are focused on location of fins/inserts/bifurcation plates, still there is no clear conclusion on the optimum shape of inserts with same length. The numerical analysis is divided into two stages; (i) To use different solid inserts in the middle of conventional plane microchannel and the performance of these microchannels are compared with the plane channel to find the optimum solid insert. (ii) Further, the improvement in combined hydrothermal performance of microchannel with solid insert is achieved by modifying the design of best performing insert.

Large wake region is observed when the insert is rectangular shaped with two sharp corners (MC-B and D). When flow velocity increases, the inserts with sharp corners generate flow mixing through flow disturbance thus increases heat transfer (Fig. 5). Microchannel with rectangular insert shows better heat transfer performance with 35 to 45% more compared to plane channel. The overall performance factor shows that the channel with convergent rectangular insert (MC-C) has 15 to 25% improvement in overall performance for the range of  $Re$  from 500 to 1000 and this configuration is superior among all seven configurations. Total thermal resistance (Eqn. 6) is the sum of conduction (Eqn. 3), convective (Eqn. 4) and capacitive thermal resistance (Eqn. 5). Convective thermal resistance is the controlling resistance among the three resistances which is around 90% of total thermal resistance. The contribution of capacitive thermal resistance in overall thermal resistance will be significantly low when high volumetric heat capacity ( $\rho \times c_p$ ) coolant used in microchannel heat sink. Water is a good choice for reducing capacitive thermal resistance Tuckerman and Pease (1981). For example, from the analysis of microchannel with vertical bifurcation, the contribution of individual thermal resistances is shown in Fig. 6. The overall performance of proposed microchannel configuration is calculated from entropy generation minimization principle Bejan (1996). The entropy generation rate due to pressure drop (Eqn. 7) and heat transfer (Eqn. 8).

$$R_{cond} = \left(\frac{t_w}{k_s A_b}\right) \quad (3)$$

$$R_{conv} = \left(\frac{1}{h_{avg} A_h}\right) \quad (4)$$

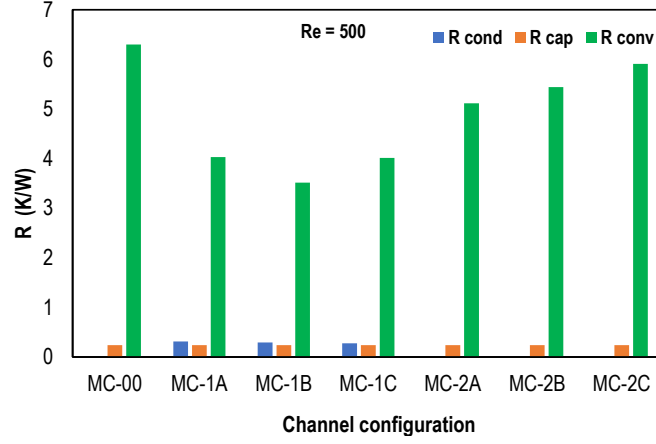


Figure 6: Effect of Bifurcation on Thermal Resistances (MC-00 to MC-2C Represents Different Flow Bifurcation Microchannels)

$$R_{cap} = \left( \frac{1}{2\dot{m}c_p} \right) \quad (5)$$

$$R_{th} = R_{cond} + R_{conv} + R_{cap} \quad (6)$$

$$S_{\Delta P} = \frac{\dot{m}\Delta P}{\rho T_{in}} \quad (7)$$

$$S_{\Delta T} = \frac{Q(T_{w,avg} - T_{in})}{T_{w,avg} T_{in}} \quad (8)$$

To extend the analysis of microchannel with solid fins, the numerical study has been conducted by replacing the solid insert by porous insert. Six microchannel configurations are studied by varying the location of the porous fin along the length of the channel for various flow Re. Further, the optimized location is selected to study the effect of porous fin thickness, porosity, permeability and quadratic drag factor on the pressure drop and heat transfer coefficient values. The present porous work shows how the slip flow and boundary layer redevelopment contributes for the effective heat transfer enhancement. At the interface of solid fin and the fluid, the velocity will be zero due to no-slip boundary condition. But fluid can flow through porous media or permeate into the porous media when the solid fin is replaced by porous fin. The permeation of the fluid can cause non-zero velocity distribution at the fluid-porous interface. This behavior shows slip of fluid on the porous fin wall and this is the reason for the total pressure drop reduction in the microchannel heat sink. To quantify this reduction in pressure drop, Chuan *et al.* (2015) mentioned the relation between pressure drop and slip length ( $\delta$ ) as given in Eqn. (9). From Fig. 7, it is clearly mentioned that how the slip length can be estimated from the velocity distribution when the porous fin is placed in the channel path. The larger the slip length, more reduction in pressure drop is observed.

$$\frac{\Delta p_{no-slip} - \Delta p_{slip}}{\Delta p_{no-slip}} = \frac{3\delta}{\frac{H_{ch}}{2} + 3\delta} \quad (9)$$

Some of the key observations from the porous fin analysis are given below;

- The optimum location of the porous fin is identified as 6.2 mm away from the channel inlet shows superior performance than other positions.
- The total pressure drop values increases with fin porosity. Higher fin porosity shows poor heat transfer performance due to lower convective surface area.
- The permeability value plays a significant role in heat transfer enhancement in the porous medium among all porous parameters. The average heat transfer coefficient decreases with increasing permeability.
- The effect of drag factor on overall performance is not significant.

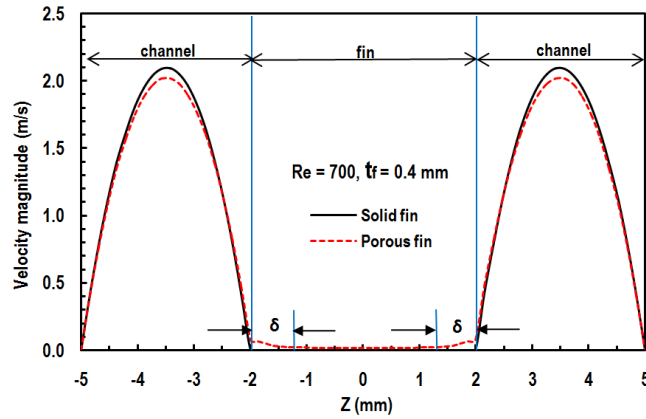


Figure 7: Velocity Distribution in Channel and Fin (Solid, Porous); Slip length ( $\delta$ ) Estimation in Porous Fin

### 4.3 Microchannel Heat Sink for Partly Heated Region Applications

Electronic components generate heat at different regions; such conditions are taken into account in this work by imposing local heat flux at the crest or trough of the wavy bottom wall. In order to study the thermal performance of the microchannel in practical applications, the complete bottom wavy wall has been imposed with four different thermal boundary conditions. The first two are the usual boundary conditions, (1) uniform heat flux and (2) constant wall temperature conditions imposed on the bottom wall to study the effect of waviness. To study the effect of partly heated region applications, the other two boundary conditions imposed are (3) heat flux at crest and (4) heat flux at trough. For better clarity, Fig. 8 shows the axial variation of  $Nu$  for a small portion of the channel length (30–50 mm). The wavy channel MC-02 ( $\lambda = 12D_h$  and  $A = 0.2D_h$ ) attains a larger peak of  $Nu$  values along the axial direction. Such locations would be preferable to absorb more heat. In practical applications, when the microchannel serves the purpose of heat sink, it is observed that the crests of the wavy wall should match with the heat sources.

## 5 Conclusions

The hydro-thermal performance of microchannel heat sink is analyzed by introducing surface modifications, solid and porous inserts in the flow path. Flow and heat transfer characteristics were investigated separately and the overall performance of proposed microchannel configurations were compared with simple microchannel. The key observations from the overall numerical analysis are given below.

- Convective thermal resistance is the dominant factor (around 90%) contributing to overall thermal resistance.
- The entropy generation rate due to heat transfer is larger compared to the entropy generation rate due to fluid flow friction.
- Introducing of local waviness in microchannels with appropriate amplitude and wavelengths eliminates hot spots and results in uniform surface temperature.
- It was observed that the proposed microchannels with single insert has better heat transfer performance with minimum increase in pressure drop.
- The incorporation of porous fin in the microchannel leads to reduction in channel pressure drop due its slip effect. The permeability value plays a significant role in heat transfer enhancement.



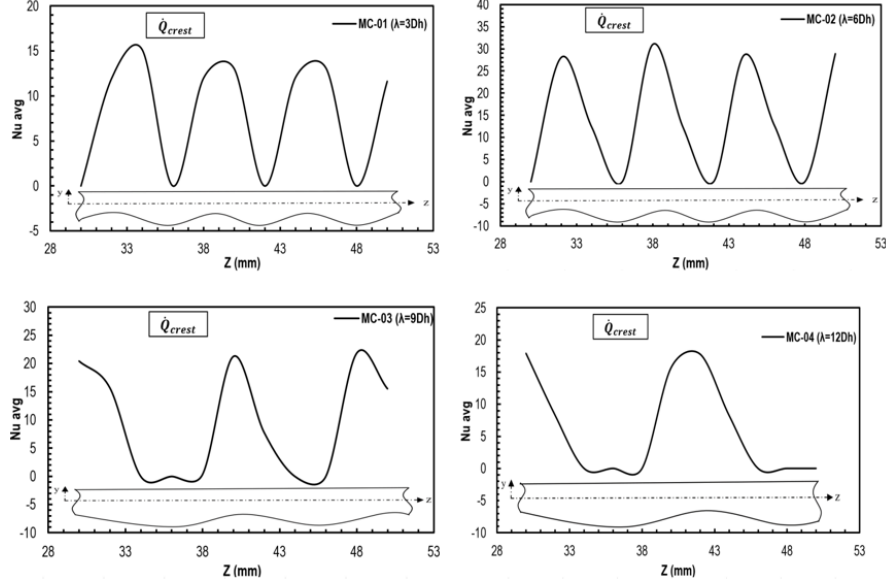


Figure 8: Axial Nusselt Number Variation in Wavy Microchannels at  $Re = 500$

- The water + CuO nano-fluid produces larger pressure drop among chosen working fluids. The percentage decrement in overall thermal resistances for water +  $Al_2O_3$  nanofluid is about 6 to 7% while compared with pure water.
- It is found that in practical applications, when the microchannel serves the purpose of heat sink, the crests of the wavy wall should match with the heat sources.

## 6 Organization of the Thesis

The proposed outline of the thesis is as follows:

**Chapter 1** Introduction

**Chapter 2** Literature Review

**Chapter 3** Objectives and Methodology

**Chapter 4** Heat Transfer Enhancement in Microchannel Heat Sink due to Waviness in Channel Wall

**Chapter 5** Heat Transfer Enhancement in Microchannel Heat Sink due to Channel Bifurcation

**Chapter 6** Hydro-Thermal Analysis of Microchannel Heat Sink with Solid and Porous Inserts

**Chapter 7** Microchannel Application for Discrete Heat Source Applications

**Chapter 8** Conclusions

## Nomenclature

$A$  Wave amplitude, m

$A_b$  Base area,  $m^2$

$A_h$  Convective area,  $m^2$

$AR$  Aspect ratio

$c_p$  Specific heat,  $J/kgK$

$D_h$  Hydraulic diameter, m

$f$  Fanning friction factor

$T$  Temperature, K

$t_w$  Bottom wall thickness, m

$x, y, z$  Co-ordinate axes

### Greek Symbols

$\Delta P$  Total channel pressure drop,  $N/m^2$

$\lambda$  Wavelength, m

$\delta$  Slip length, m

$H_{ch}$	Microchannel height, m	<b>Suffix</b>
$h$	Heat transfer coefficient, $W/m^2K$	<i>avg</i> Average
$k_s$	Solid thermal conductivity, $W/mK$	<i>ch</i> Channel
$\dot{m}$	Mass flow rate, $kg/s$	<i>cap</i> Capacitive
$Nu$	Nusselt number	<i>cond</i> Conduction
$PI$	Performance Index	<i>conv</i> Convection
$R$	Thermal resistance, $K/W$	<i>th</i> Total thermal
$Re$	Reynolds number	0 Plane channel
$S$	Entropy generation rate, $W/K$	in Inlet

## 7 List of Publications

### 7.1 Journal Publications (SCI Index)

- (a) **Kumar, D.S** and Jayavel, S., 2021. Microchannel with Waviness at Selective Locations for Liquid Cooling of Microelectromechanical Devices. *Journal of Applied Fluid Mechanics*, 14(3).
- (b) **Kumar, D.S** and Jayavel, S., 2021. Optimization of porous fin location and investigation of porosity and permeability effects on hydro-thermal behavior of rectangular microchannel heat sink. *International Communications in Heat and Mass Transfer*, 129, p.105737.
- (c) **Kumar, D.S** and Jayavel, S., 2022. Effect of Wavy Wall and Plate Bifurcations on Heat Transfer Enhancement in Microchannel. *Journal of Electronic Packaging*, 144(4).
- (d) **Kumar, D.S** and Jayavel, S., 2022. Effect of Location of Discrete Heat Sources on a Wavy-Wall Microchannel for Liquid Cooling. *Journal of Enhanced Heat Transfer*, 29(2).

### 7.2 Book Chapters

- (a) **D. Sathish Kumar** and S. Jayavel, Effect of Channel Confinement and Hydraulic Diameter on Heat Transfer in a Micro-channel, *Lecture Notes in Mechanical Engineering*, Select Proceedings of NHTFF 2018.
- (b) **D. Sathish Kumar** and S. Jayavel, Numerical Analysis of Smooth and Wavy Wall Microchannel Heat Sink for Electronic Cooling Applications, In *Recent Advances in Computational and Experimental Mechanics*, Vol—I, pp. 299-309. Springer, Singapore, 2022.

### 7.3 Conference Publications

- (a) **D. Sathish Kumar** and S. Jayavel, Effect of Channel Confinement and Hydraulic Diameter on Heat Transfer in a Microchannel, *Proceedings of International Conference on Numerical Heat Transfer and Fluid Flow (NHTFF-2018)*, NIT Warangal, January 19-21, 2018.
- (b) **D. Sathish Kumar** and S. Jayavel, Numerical Study of Multi Microchannel Heat Sink for Electronic Cooling, 25th National and 3rd International ISHMT-ASTFE Heat and Mass Transfer Conference (IHMTTC-2019), Dec 2019, IIT Roorkee.
- (c) **D. Sathish Kumar** and S. Jayavel, Numerical Analysis of Smooth and Wavy Wall Microchannel Heat Sink for Electronic Cooling Applications, *Proceedings of 1st Online International Conference on Recent Advances in Computational and Experimental Mechanics (ICRACEM - 2020)*, September 4-6, 2020, IIT Kharagpur.

- (d) **D. Sathish Kumar** and S. Jayavel, Effect of Substrate Thickness and Material on Wavy Wall Microchannel Heat Sink for Electronic Cooling Applications, Proceedings of the 48th National Conference on Fluid Mechanics and Fluid Power (FMFP) December, 2021, BITS Pilani, Pilani Campus, Rajasthan, India.
- (e) **D. Sathish Kumar** and S. Jayavel, Numerical Analysis of Nano Fluid Based Wavy Wall Microchannel Heat Sink for Electronic Cooling Applications, Proceedings of the 26th National and 4th International ISHMT-ASTFE Heat and Mass Transfer Conference December 17-20, 2021, IIT Madras, India.

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