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Synopsis Of

The Complexity of Star Colouring and its Relatives

A Thesis

To be submitted by

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DOCTOR OF PHILOSOPHY

1 Abstract

Star colouring, restricted star colouring (abbreviated rs colouring) and acyclic colouring are variants of graph colouring used as models in the computation of sparse Hessian matrices [12, 13]. We study the complexity of these three colouring variants (i) in well-known graph classes such as planar graphs and bipartite graphs, and (ii) with respect to the maximum degree of the graph, focusing on graphs of maximum degree d and d-regular graphs. For a fixed $k \in \mathbb{N}$, the problem k-Star Colourability takes a graph G as input, and asks whether G admits a k-star colouring. The problems k-RS Colourability and k-Acyclic Colourability are defined likewise.

Note that 3-Star Colourability is known to be NP-complete in (i) planar bipartite graphs [1], (ii) graphs of maximum degree 4 [15], and (iii) graphs of arbitrarily large girth [2]; we show that it is NP-complete in the intersection of the above three classes. Other results we prove include NP-completeness of 4-Star Colourability in planar 4-regular graphs, NP-completeness of k-RS Colourability in planar bipartite graphs for each $k \geq 3$, an inapproximability result on rs colouring, and an $O(n^3)$-time algorithm to test whether a chordal graph is 3-rs colourable.

For graph-theoretic problems, the complexity in relation to one fixed graph parameter is attracting attention recently (see [5, 17]). Brause et al. [5] studied the complexity of 3-Star Colourability and 3-Acyclic Colourability with respect to the diameter of the graph. We study the complexity of k-Star Colourability, k-RS Colourability and k-Acyclic Colourability with respect to the maximum degree of the graph for $k \geq 3$.

Let us consider graphs of maximum degree d. For $k \geq 3$, there exists a $d^* \in \mathbb{N}$ such that k-Star Colourability in graphs of maximum degree d^* is NP-complete (e.g.: we prove this for $d^* = k$). Also, if k-Star Colourability is NP-complete for graphs of maximum degree d, then it is NP-complete for graphs of maximum degree $d+1$. Hence, there exists a $d_0 \in \mathbb{N}$ such that k-Star Colourability in graphs of maximum degree d is NP-complete for all $d \geq d_0$. The same applies to k-RS Colourability and k-Acyclic Colourability. We study the least integer d such that k-Star Colourability (resp. k-RS Colourability or k-Acyclic Colourability) is NP-complete for graphs of maximum degree d, and denote it by $\widetilde{L}_s^{(k)}$ (resp. $\widetilde{L}_{rs}^{(k)}$ or $\widetilde{L}_a^{(k)}$). From reductions in the literature [6, 7], it follows that $k(k-1+\lceil\sqrt{k}\,\rceil)$ is an upper bound for $\widetilde{L}_s^{(k)}$ and $\widetilde{L}_a^{(k)}$. We prove linear upper bounds on $\widetilde{L}_s^{(k)}$, $\widetilde{L}_{rs}^{(k)}$ and $\widetilde{L}_a^{(k)}$: (i) $\widetilde{L}_s^{(3)} = 3$ and $\widetilde{L}_s^{(k)} \leq k$ for $k \geq 4$, (ii) $\widetilde{L}_{rs}^{(3)} = 3$ and $\widetilde{L}_{rs}^{(k)} \leq k-1$ for $k \geq 4$, and (iii) $\widetilde{L}_a^{(k)} \leq k+1$ for $k \geq 3$. For $k = 5$ and $k \geq 7$, we improve the upper bound on $\widetilde{L}_s^{(k)}$ to $k-1$.

Let us shift our focus to the class of d-regular graphs. In this thesis, we show that for each $k \in \mathbb{N}$, there exist $d_1, d_2 \in \mathbb{N}$ such that k-Star Colourability (resp. k-RS Colourability or k-Acyclic Colourability) in d-regular graphs is polynomial-time solvable whenever $d \leq d_1$ or $d \geq d_2$. Hence, if k-Star Colourability in d^*-regular graphs is NP-complete for some $d^* \in \mathbb{N}$, then there exists a least integer d (resp. highest integer d) such that k-Star Colourability in d-regular graphs is NP-complete, which is denoted by $L_s^{(k)}$ (resp. $H_s^{(k)}$).

The definitions of $L_{rs}^{(k)}$, $H_{rs}^{(k)}$, $L_a^{(k)}$ and $H_a^{(k)}$ are similar. We show that (i) for $k = 5$ and $k \geq 7$, $L_s^{(k)} = \widetilde{L}_s^{(k)}$ and $H_s^{(k)} \leq 2k - 4$, (ii) for $k \geq 4$, $L_{rs}^{(k)} = \widetilde{L}_{rs}^{(k)}$ and $H_{rs}^{(k)} = k - 1$; and (iii) for $k \geq 4$, $L_a^{(k)} = \widetilde{L}_a^{(k)}$ and $H_a^{(k)} = 2k - 3$. We conjecture that $H_s^{(k)} = 2k-4$ for $k \geq 4$, and prove this for $k = 4$. For $k \geq 4$, $H_s^{(k)} = 2k-4$ if and only if k-Star Colourability is NP-complete for $(2k-4)$-regular graphs. We study the structure of $(2k-4)$-regular k-star colourable graphs with $k \geq 4$, or equivalently $2p$-regular $(p+2)$-star colourable graphs with $p \geq 2$. For every $d \geq 2$ and every d-regular graph G, we prove a lower bound of $\lceil (d+4)/2 \rceil$ colours to star colour G. Hence, for $p \geq 2$, at least $p + 2$ colours are required to star colour a $2p$-regular graph G; that is, $2p$-regular $(p+2)$-star colourable graphs form a class of extremal graphs with respect to our lower bound. We characterise this class of extremal graphs in terms of (i) bicoloured components, (ii) graph orientations, and (iii) locally constrained graph homomorphisms. For instance, we show that for each $p \geq 2$, a $2p$-regular graph G admits a $(p+2)$-star colouring if and only if G admits a $(p+2)$-colouring such that every bicoloured component is isomorphic to $K_{1,p}$. For $p \geq 2$, we show a number of properties of $2p$-regular $(p+2)$-star colourable graphs G including the following : (i) $|V(G)|$ is divisible by $(p+1)(p+2)$, (ii) G is (diamond, K_4)-free, (iii) $\chi(G) \leq 3\log_2(p+2)$, and (iv) if G is $K_{1,p+1}$-free, then G is a clique graph. For $p \geq 2$, we construct a $2p$-regular $(p+2)$-star colourable vertex-transitive graph G_{2p} and a $2p$-regular $(p+2)$-star colourable Hamiltonian graph on $t(p+1)(p+2)$ vertices for each $t \in \mathbb{N}$. We prove that a $K_{1,p+1}$-free $2p$-regular graph G with $p \geq 2$ is $(p+2)$-star colourable if and only if G admits a locally bijective homomorphism to G_{2p}. Moreover, for every 3-regular graph G, the line graph of G is 4-star colourable if and only if G is bipartite and distance-two 4-colourable.

2 Objectives

- To study the complexity of k-Star Colourability, k-RS Colourability and k-Acyclic Colourability
  - (a) in well-known graph classes such as planar graphs and bipartite graphs,
  - (b) in graphs of maximum degree d and d-regular graph, focusing on the values of $\widetilde{L}_s^{(k)}$, $L_s^{(k)}$, $H_s^{(k)}$, $\widetilde{L}_{rs}^{(k)}$, $L_{rs}^{(k)}$, $H_{rs}^{(k)}$, $\widetilde{L}_a^{(k)}$, $L_a^{(k)}$ and $H_a^{(k)}$.
- For each $p \geq 2$, characterise $2p$-regular $(p+2)$-star colourable graphs.

3 Existing Gaps Which were Bridged

Analysing the boundary between easy (i.e., polynomial-time solvable) and hard (e.g., NP-complete) problems is a common theme in complexity theory [11]. Studying the change in the complexity of a problem in response to a change in a single parameter falls in this category. Brause et al. [5] studied the complexity of 3-Star Colourability and 3-Acyclic Colourability with the diameter of the graph as the parameter. A similar investigation of the complexity of colouring variants such as star colouring with the maximum degree as the parameter has not been done before (to the best of our knowledge). We investigate the complexity of k-Star Colourability, k-RS Colourability and k-Acyclic Colourability with the maximum degree as the parameter.

We also studied the complexity of star colouring, rs colouring acyclic colouring in some well-known graph classes. The problem Star Colourability takes a graph G and a positive integer k as input, and asks whether G admits a k-star colouring. The problems RS Colourability and Acyclic Colourability are defined likewise. Table 1 and Table 2 give an overall picture of their complexity (an entry '?' indicates that the status is unknown). In both tables, our results are highlighted.

Table 1: Complexity of Acyclic, Star and RS Colourability.

Graph class	Acyclic Colourability	Star Colourability	RS Colourability
Planar	NPC [19]	NPC [1]	**NPC**
Bipartite	NPC [6]	NPC [7]	**NPC**
Co-bipartite	?	**NPC**	**NPC**
Split	P [14]	?	**P**
Cograph	P [16]	P [16]	**P**
Planar $\cap$ girth ≥ 7	P [4]	**NPC**	**NPC**

Table 2: Complexity of k-Acyclic Colourability, k-Star Colourability and k-RS Colourability for $k \geq 3$.

Graph class	k-Acyclic Colourability	k-Star Colourability	k-RS Colourability
Bipartite	NPC [6]	NPC [7]	**NPC**
Co-bipartite	P [2]	**P**	**P**
Chordal	P [14]	?	**P** for $k = 3$? for $k \geq 4$
Planar	NPC for $k \leq 4$ [19] P for $k \geq 5$ [3]	NPC for $k = 3$ [1] **NPC** for $k = 4$? for $5 \leq k \leq 19$ P for $k \geq 20$ [1]	**NPC**

We proved the first NP-completeness and inapproximability results on rs colouring (an open problem in [20]), and improved a number of results on star colouring and acyclic colouring as shown below.

(a) To star colour a 3-regular graph, at least 4 colours are needed [21], and to star colour a d-regular graph, at least $\lceil (d+3)/2 \rceil$ colours are needed [9] (this bound is proved for the hypercube Q_d in [10]).
We prove that for $d \geq 2$, at least $\lceil (d+4)/2 \rceil$ colours are required to star colour a d-regular graph, and this bound is attained for each d.

(b) 3-Star Colourability is known to be NP-complete in (i) planar bipartite graphs [1], (ii) graphs of maximum degree 4 [15], and (iii) graphs of arbitrarily large girth [2]; we show that it is NP-complete in the intersection of the above three classes (i.e., planar bipartite graphs of maximum degree 4 and arbitrarily large girth).

(c) From the reduction of Coleman and More [7] (resp. Coleman and Cai [6]), it follows that for $k \geq 3$, k-Star Colourability (resp. k-Acyclic Colourability) is NP-complete for bipartite graphs of maximum degree $k(k-1+\lceil\sqrt{k}\rceil)$; we bring down the maximum degree from $k(k-1+\lceil\sqrt{k}\rceil)$ to k (resp. $k+1$).

(d) Dvorák et al. [8] proved that for every 3-regular graph G, the line graph of G is 4-star colourable if and only if G admits a locally bijective homomorphism to the hypercube Q_3. We prove that for every 3-regular graph G, the line graph of G is 4-star colourable if and only if G is bipartite and distance-two 4-colourable. Moreover, we put their result in a different perspective by showing that an arbitrary $K_{1,p+1}$-free $2p$-regular graph G is $(p+2)$-star colourable if and only if G admits a locally bijective homomorphism to G_{2p} (this is interesting because line graphs are $K_{1,3}$-free, and $G_4 \cong L(Q_3)$).

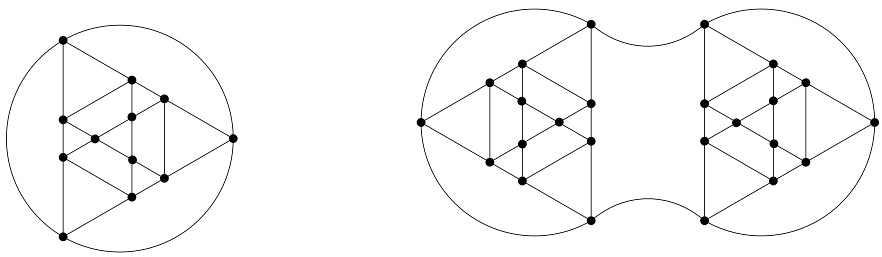
(e) Ochem [19] proved that 3-Acyclic Colourability is NP-complete for bipartite graphs of maximum degree 4. Mondal et al. [18] proved that 4-Acyclic Colourability is NP-complete for graphs of maximum degree 5.
We prove that for $k \geq 3$, k-Acyclic Colourability is NP-complete for bipartite graphs of maximum degree $k+1$, thereby generalising the result of Ochem, and adding bipartiteness to the result of Mondal et al.

4 Most Important Contributions

(a) For $d \geq 3$, we prove that $\chi_s(G) \geq \lceil (d+4)/2 \rceil$ for every d-regular graph G, and we characterise even-degree regular graphs that attain this bound in terms of (i) bicoloured components, (ii) graph orientations, and (iii) locally constrained graph homomorphisms.

(b) For $p \geq 2$, $2p$-regular $(p+2)$-star colourable graphs G have the following properties: (i) the number of vertices in G is divisible by $(p+1)(p+2)$, (ii) G does not contain diamond or circular ladder graph CL_{2r+1} as a subgraph for any $r \in \mathbb{N}$, (iii) $\alpha(G) > |V(G)|/4$, and (iv) $\chi(G) \leq 3\log_2(p+2)$.

(c) For $p \geq 2$, we construct a $2p$-regular $(p+2)$-star colourable vertex-transitive graph G_{2p}. For $p \geq 2$ and $t \in \mathbb{N}$, we construct a $2p$-regular $(p+2)$-star colourable Hamiltonian graph on $t(p+1)(p+2)$ vertices; for $p = 2$, the graphs constructed are also planar (see Figure 1).



(a) For $t = 1$ (b) For $t = 2$

Figure 1: First two members of a family of planar 4-regular 4-star colourable Hamiltonian graphs on $12\,t$ vertices.

(d) Let $p \geq 2$, and let G be a $K_{1,p+1}$-free $2p$-regular graph. We prove that G is $(p+2)$-star colourable if and only if G admits a locally bijective homomorphism to G_{2p}. If G is $(p+2)$-star colourable, then $G[N_G(v)] \cong pK_2$ for every vertex v of G, and thus G is a clique graph and $K(K(G)) \cong G$.

(e) 3-Star Colourability is NP-complete for planar bipartite graphs of maximum degree 3 and arbitrarily large girth.

(f) Given a graph G, it is coNP-hard to test whether G has a unique 3-star colouring up to colour swaps [1].

(g) Let k-Star Colourability(bipartite, max. deg. k) denote the restriction of k-Star Colourability to the class of bipartite graphs of maximum degree k. For $k \geq 3$, k-Star Colourability(bipartite, max. deg. k) is NP-complete, and the problem does not even admit a $2^{o(n)}$-time algorithm unless Exponential Time Hypothesis (ETH) fails. Hence, $\widetilde{L}_s^{(k)} \leq k$ for $k \geq 3$.

(h) For every 3-regular graph G, the line graph of G is 4-star colourable if and only if G is bipartite and distance-two 4-colourable.
As a result, 4-Star Colourability is NP-complete for planar 4-regular graphs, and thus $H_s^{(4)} = 4$.

(i) For $k = 5$ and $k \geq 7$, k-Star Colourability is NP-complete for graphs of maximum degree $k-1$, and as a result $\widetilde{L}_s^{(k)} \leq k-1$ and $L_s^{(k)} = \widetilde{L}_s^{(k)}$.

(j) For each $k \geq 3$, it is NP-complete to test whether a k-star colourable graph is k-rs colourbale.

(k) For the class of co-bipartite graphs, Star Colourability and RS Colourability are NP-complete, whereas k-Star Colourability and k-RS Colourability are polynomial-time solvable for each k.

(l) An $O(n)$-time algorithm to decide 3-RS Colourability in trees, and an $O(n^3)$-time algorithm to decide 3-RS Colourability in chordal graphs.

[1] G has a unique 3-star colouring up to colour swaps if (i) G admits a 3-star colouring f_1, and (ii) for every 3-star colouring f_2 of G, there exists a permutation σ of colours such that $f_2(v) = \sigma(f_1(v))$ for every vertex v of G

(m) For $k \geq 3$, k-RS Colourability is NP-complete for planar bipartite graphs of maximum degree k and arbitrarily large girth. As a result, $\widetilde{L}_{rs}^{(k)} \leq k$ for $k \geq 3$, and thus $\widetilde{L}_{rs}^{(3)} = 3$.

(n) 4-RS Colourability is NP-complete for planar graphs of maximum degree 3 and girth 5.

(o) For $k \geq 4$, k-RS Colourability is NP-complete for triangle-free graphs of maximum degree $k-1$, and thus $\widetilde{L}_{rs}^{(k)} \leq k-1$.

(p) For $k \geq 4$, we have $L_{rs}^{(k)} = \widetilde{L}_{rs}^{(k)}$ and $H_{rs}^{(k)} = k-1$.

(q) For all $\epsilon > 0$, it is NP-hard to approximate the problem of rs colouring a given graph with the minimum number of colours (i.e., Min RS Colouring) within $n^{\frac{1}{3}-\epsilon}$, where n is the number of vertices in the input graph.

(r) Let k-Acyclic Colourability (bipartite, max. deg. $k+1$) denote the restriction of k-Acyclic Colourability to the class of bipartite graphs of maximum degree $k+1$. For $k \geq 3$, k-Acyclic Colourability(bipartite, max. deg. $k+1$) is NP-complete, and the problem does not even admit a $2^{o(n)}$-time algorithm unless ETH fails. Hence, $\widetilde{L}_{a}^{(k)} \leq k+1$ for $k \geq 3$.

(s) For $k \geq 4$, we have $L_{a}^{(k)} = \widetilde{L}_{a}^{(k)}$ and $H_{a}^{(k)} = 2k-3$.

(t) For $k \geq 3$, given a graph G, it is coNP-hard to test whether G admits a unique k-acyclic colouring up to colour swaps and automorphisms [2].

5 Conclusions

We studied the complexity of star colouring, rs colouring and acyclic colouring in some well-known graph classes such as planar graphs and bipartite graphs, and improved some known NP-completeness results. In particular, we studied their complexity with respect to the maximum degree of the graph. For $k \geq 3$, we established linear upper bounds (close to k) on $\widetilde{L}_{s}^{(k)}$, $\widetilde{L}_{rs}^{(k)}$ and $\widetilde{L}_{a}^{(k)}$. For the class of regular graphs, we arrive at the following conclusion. Although there exist integers k and d such that the complexity of k-Star Colourability in graphs of maximum degree d differ from that in d-regular graphs, study of $L_{s}^{(k)}$ is tied to the study of $\widetilde{L}_{s}^{(k)}$ (see Result (i) in Section 4). The same is true for rs colouring and acyclic colouring (see Results (p) and (s) in Section 4). An important consequence of our results is that finding the value of $\widetilde{L}_{rs}^{(k)}$ (resp. $\widetilde{L}_{a}^{(k)}$) suffices to characterize the values of d for which k-RS Colourability (resp. k-Acyclic Colourability) in d-regular graphs is NP-complete. Formally, for $k \geq 4$, k-RS Colourability (resp. k-Acyclic Colourability) in d-regular graphs is NP-complete if and only if $\widetilde{L}_{rs}^{(k)} \leq d \leq k-1$ (resp. $\widetilde{L}_{a}^{(k)} \leq d \leq 2k-3$). It is unknown whether a similar

[2] G has a unique k-acyclic colouring up to colour swaps and automorphisms if (i) G admits a k-acyclic colouring f_1, and (ii) for every k-acyclic colouring f_2 of G, there exists a permutation σ of colours and an automorphism ψ of G such that $f_2(\psi(v)) = \sigma(f_1(v))$ for every vertex v of G

result holds for k-Star Colourability as well. The following are some other open problems related to this work.

(a) For each $k \geq 3$, find $\widetilde{L}_s^{(k)}$, $\widetilde{L}_{rs}^{(k)}$ and $\widetilde{L}_a^{(k)}$.

(b) For $k \in \{4, 6\}$, is k-Star Colourability NP-complete for graphs of maximum degree $k - 1$?

(c) For $p \geq 2$, is $(p + 2)$-Star Colourability NP-complete for $2p$-regular graphs?
(We proved this for $p = 2$)

(d) For $p \geq 2$, characterise $(2p - 1)$-regular $(p + 2)$-star colourable graphs.

For $p \geq 2$, we characterised $2p$-regular $(p + 2)$-star colourable graphs in terms of (i) bicoloured components, (ii) graph orientations, and (iii) locally constrained graph homomorphisms. To this end, we introduced two notions called out-neighbourhood bijective homomorphisms and colourful Eulerian orientations. Studying the properties of out-neighbourhood bijective homomorphisms and comparing them to the properties of locally bijective homomorphisms is an important future direction. We proved that the diamond graph does not admit any colourful Eulerian orientation. Characterisation of graphs that do not admit any colourful Eulerian orientation is an interesting future direction.

6 Organization of the Thesis

The proposed outline of the thesis is as follows:

(a) Chapter 1: Preliminaries

(b) Chapter 2: Introduction and Important Notions

(c) Chapter 3: Restricted Star Colouring

(d) Chapter 4: Star Colouring of Bounded Degree Graphs and Regular Graphs

(e) Chapter 5: Hardness Transitions of Star Colouring

(f) Chapter 6: Acyclic Colouring

(g) Chapter 7: Conclusion

7 List of Publications

7.1 Journal Publications

(a) Shalu M. A., and Cyriac Antony. " The complexity of restricted star colouring", *Discrete Applied Mathematics* (SCI), 319, 327–350, (2022), Elsevier, doi: 10.1016/j.dam.2021.05.015.

(b) Shalu M. A., and Cyriac Antony. " Star colouring of bounded degree graphs and regular graphs", *Discrete Mathematics* (SCI), 345 (6), 112850, (2022), Elsevier, doi: 10.1016/j.disc.2022.112850.

7.2 Conference Publications

(a) Shalu M. A., and Cyriac Antony, Complexity of restricted variant of star colouring, *Conference on Algorithms and Discrete Applied Mathematics (CALDAM 2020)*, 3–14, (2020), doi: 10.1007/978-3-030-39219-2_1.

(b) Shalu M. A., and Cyriac Antony, The complexity of star colouring in bounded degree graphs and regular graphs, *Conference on Algorithms and Discrete Applied Mathematics (CALDAM 2022)*, 78–90, (2022), doi: 10.1007/978-3-030-95018-7_7.

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